ISSN 0018-151X, High Temperature, 2010, Vol. 48, No. 1, pp. 140–143. © Pleiades Publishing, Ltd., 2010.
Original Russian Text © V.R. Pesochin, 2010, published in Teplofizika Vysokikh Temperatur, 2010, Vol. 48, No. 1, pp. 148–151.

BRIEF COMMUNICATIONS

Excitation of Acoustic Vibrations under Conditions of Condensation of Moist Steam in a Heated Channel

V. R. Pesochin

Joint Institute for High Temperatures, Russian Academy of Sciences (IVTAN), Moscow, 125412 Russia

Received February 16, 2009

DOI: 10.1134/S0018151X10010190

INTRODUCTION

Moist steam is employed nowadays in numerous industrial processes. For example, moist steam is employed in the last stages of steam turbines as working medium. Therefore, much attention is given to studying the motion of moist steam in view of phase transformations [1, 2]. Previously [3, 4], the conditions of excitation of acoustic vibrations were studied in the case of condensation of silica in the loop of an open-cycle MHD generator. These investigations were performed in a linear formulation using the method of separation of variables in view of the respective boundary conditions. In so doing, the classical theory was used in [3] for describing the process of nucleation. In [4], the process of condensation was described in an equilibrium approximation. The present study deals with the investigation of the conditions of excitation of acoustic vibrations in moist steam during its flow in a heated channel of cylindrical shape. This investigation was performed in an equilibrium approximation and in a linear formulation.

BASIC EQUATIONS

The flow in the channel was taken to be one-dimensional under conditions of thorough stirring in the transverse direction and of the absence of stirring in the longitudinal direction. When considering acoustic vibrations, moist steam was taken to be ideal gas, and the presence of dispersed phase was ignored. The droplets of liquid were taken to be spherical and monodisperse. The specific heat capacity of steam was taken to be constant and independent of temperature. Such assumptions are standard in calculations of the processes which occur in power plants [1, 2]. In addition, it was assumed that the channel is heated by a uniform heat flux.

The equations of continuity, motion, and energy may be written as

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial y}(\rho u)=-W; \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}; \quad (2)$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial y}\right)=LW+\frac{\partial p}{\partial t}+\frac{1}{R}\frac{\partial}{\partial R}(qR). \quad (3)$$

Here, W is the mass rate of condensation per unit volume, L is the heat of condensation, R is the current radius of the channel, and q is the heat flux from the channel wall.

We will further assume that local thermodynamic equilibrium is maintained in moist steam. This assumption is common in the theory of evaporation of droplets [5]. Then the equation of state may be written as

$$\rho=\rho(p,T). \quad (4)$$

It is known from experience gained in developing combustors of gas-turbine plants and liquid-propellant rocket engines [5, 6] that the most dangerous are high-frequency acoustic vibrations. Therefore, this study involved an investigation of excitation of high-frequency acoustic vibrations in moist steam under conditions of phase transitions. The high-frequency acoustic vibrations are characterized by $\mathrm{Sh} \gg 1$ [6], where Sh is the Strouhal number ($\mathrm{Sh}=\frac{\omega l}{u}$, ω is the circular vibration frequency, and l is the channel length). We linearize Eqs. (1)–(3) in view of $\mathrm{Sh} \gg 1$ to derive

$$\frac{\partial\rho'}{\partial t}+\rho\frac{\partial u'}{\partial y}+u'\frac{\partial\rho}{\partial y}=-W', \quad (5)$$

$$\frac{\partial u'}{\partial t}=-\frac{1}{\rho}\frac{\partial p'}{\partial y}, \quad (6)$$

$$\rho c_p\left(\frac{\partial T'}{\partial t}+u'\frac{\partial T}{\partial y}\right)=LW'+\frac{\partial p'}{\partial t}+\frac{1}{R}\frac{\partial}{\partial R}(q'R). \quad (7)$$

The primes indicate perturbations.

We linearize Eq. (4) to derive

$$\frac{\rho'}{\rho}=\frac{p'}{\rho a_T^2}-\frac{T'}{T},$$

where a_T is the isothermal velocity of sound.

We substitute the latter expression into Eq. (5) in view of (7) to derive

$$\frac{1}{\gamma}\frac{\partial}{\partial t}\left(\frac{p'}{p}\right)+\frac{\partial u'}{\partial y}=\frac{L}{c_pT}\frac{W'}{\rho}+\frac{1}{T\rho c_p}\frac{1}{R}\frac{\partial}{\partial R}(q'R). \quad (8)$$

Here, γ is the adiabatic exponent of moist steam.

We differentiate Eq. (8) with respect to t, and expression (6) with respect to y, and eliminate the perturbation of velocity u' to derive

$$\frac{\partial^2}{\partial t^2}\left(\frac{p'}{p}\right)=a_S^2\frac{\partial^2}{\partial y^2}\left(\frac{p'}{p}\right)+\frac{\gamma L}{T\rho c_p}\frac{\partial W'}{\partial t}+\frac{\gamma}{T\rho c_p}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial q'}{\partial t}\right), \quad (9)$$

where a_S is the adiabatic velocity of sound in moist steam.

In deriving the latter equation, it was taken into account that the following inequalities are usually valid:

$$\frac{1}{T\rho c_p}\frac{1}{R}\frac{\partial}{\partial R}(q'R)\ll\frac{1}{\gamma}\frac{\partial}{\partial t}\left(\frac{p'}{p}\right),\quad \frac{L}{c_pT}\ll\frac{1}{\gamma}\frac{\partial}{\partial t}\left(\frac{p'}{p}\right).$$

For solving Eq. (9), the expressions for q' and W' must be derived. The value of q' in the general case depends on the processes of heat transfer both in moist steam flowing in the channel and in the channel wall. In so doing, it is largely defined by heat transfer in moist steam [7]. Then, assuming that the thermal stabilization of turbulent flow of moist steam already occurred in the channel, we can derive [8, 9]

$$\frac{q'}{q}=\frac{\rho'}{\rho}=\frac{1}{\gamma}\frac{p'}{p}. \quad (10)$$

Here it is taken into account that, under conditions of high-frequency acoustic vibrations in the channel, the state of moist steam may be described in a quasi-adiabatic approximation [6].

In the turbulent mode of flow, the velocities of moist steam at each point in the channel cross section little differ from the cross section average velocity [8]. It is known [8] that in this case a linear distribution of heat flux along the radius takes place,

$$\frac{q}{q_l}=\frac{R}{R_0}, \quad (11)$$

where q_l is the heat flux on the channel surface, and R_0 is the inside radius of the channel.

For determining the value of W', we will consider the processes which occur in moist steam under conditions of phase transitions in acoustic wave. A pressure increase in acoustic wave will be accompanied by the rise of steam temperature; in so doing, the droplet temperature will relax to the steam temperature. This process may be described by the equation of heat balance for droplet,

$$\frac{4}{3}\pi r^3\rho_l c_p\frac{dT_l}{dt}=4\pi r\kappa(T_\infty-T_l)/(1+\theta\mathrm{Kn}).$$

Here, r is the droplet radius; ρ_l and c_l denote the density and specific heat capacity of liquid, respectively; κ is the coefficient of molecular thermal conductivity of moist steam; Kn is the Knudsen number; θ is the coefficient which is a function of Knudsen number, $\theta \sim 1$ [10]; and T_∞ and T_l denote the temperature of steam and droplet, respectively.

It follows from this equation that the characteristic time of relaxation of droplet temperature to steam temperature has the form

$$\tau^*=r^2\rho_l c_l(1+\theta\mathrm{Kn})/3\kappa.$$

The equation for heat balance for droplet is written for the case of uniform temperature field within the droplet. This is the case at $\kappa(1+\theta\mathrm{Kn})/\kappa_l\ll 1$, ($\kappa_l$ is the coefficient of thermal conductivity of liquid), which usually holds for gases and liquids.

In this study, we consider the emergence of acoustic vibrations in moist steam, the frequency of which satisfies the inequality $f\ll 1/\tau^*$.. In this case, phase transitions in acoustic field may be taken to be equilibrium. For example, for $r = 5$ μm we can have $\tau^* = 6\times10^{-4}$ s, while phase transformations in a two-phase mixture for $f\ll 1.5\times10^{-3}$ s^{-1} may be taken to be equilibrium. In the case of a polydisperse system of droplets, the spectrum of relaxation times will correspond to this system. In this case, the determination of the frequency, at which the phase transformations may be taken to be equilibrium, reduces to determining τ^* for droplets of the maximal size.

In the case of equilibrium isentropic process of moist steam, the dryness factor x is defined by the equation [2]

$$\frac{Lx}{T}=c_l\ln\left(\frac{A}{T}\right),$$

where A is the process constant.

We differentiate the latter relation with respect to time and derive

$$\frac{dx}{d\tau}=\frac{1}{T}\left(x-\frac{c_lT}{L}\right)\frac{dT}{d\tau}.$$

Here, τ is the time of isentropic process of moist steam.

It follows from the latter expression [2] that, for low values of steam quality ($x < c_lT/L$), the isentropic cooling ($dT/d\tau < 0$) causes an increase in steam quality ($dx/d\tau > 0$). We can use the Clausius–Clapeyron equation and write the latter relation as

$$\frac{dx}{d\tau}=\frac{1}{L\rho}\left(x-\frac{c_lT}{L}\right)\frac{dp}{d\tau}. \quad (12)$$

We will assume that the total mass of steam and its condensate per unit volume remains a constant quantity. This assumption is common in the case of low velocities where the lagging behind of condensate droplets may be ignored [1]. Then we derive the fol-

lowing formula using the expression for x from Eq. (12):

$$W = \frac{1}{Lx}\left(\frac{c_l T}{L} - x\right)\frac{dp}{d\tau}.$$

We will assume that the time of isentropic process τ $(\tau \sim l/u)$ is much longer than the period of acoustic vibrations. Then the expression for W' may be written as

$$W' = \frac{u}{Lx}\left(\frac{c_l T}{L} - x\right)\frac{p'}{\gamma p}\frac{dp}{dy}.$$

We substitute the latter expressions along with Eqs. (10) and (11) into Eq. (9) and derive

$$\frac{\partial^2}{\partial t^2}\left(\frac{p'}{p}\right) = a_S^2\frac{\partial^2}{\partial y^2}\left(\frac{p'}{p}\right) + 2\delta\frac{\partial}{\partial t}\left(\frac{p'}{p}\right). \quad (13)$$

Here,

$$\delta = \frac{1}{\rho c_p T}\left[\frac{u}{2x}\left(\frac{c_l T}{L} - x\right)\frac{dp}{dy} + \frac{q_l}{R_0}\right].$$

For determining the frequencies of acoustic vibrations upon their excitation, one must preassign the boundary conditions for Eq. (13). These conditions must reflect the characteristic features of the apparatus, in the channel of which phase transitions occur and acoustic vibrations are excited. In this study, we restrict ourselves to the case of channel which is acoustically open at both ends. The boundary conditions may then be written as $p' = 0$ at $y = 0$ and $p' = 0$ at $y = l$.

We solve Eq. (13) using the method of separation of variables in view of the boundary conditions and derive

$$\frac{p'}{p} = e^{\delta t}\sum_{k=1}^{\infty} B_k \sin\frac{\pi k x}{l}\cos\left(\sqrt{\omega_k^2 - \delta^2}\,t + \varphi_k\right), \quad (14)$$

where $k = 1, 2, 3 \ldots$

The values of B_k and φ_k are determined from the initial conditions. The natural frequency of steam-generating channel, determined from the boundary conditions, will be

$$\omega_k = \pi k a_S / l.$$

DISCUSSION OF THE RESULTS

One can see from Eq. (14) that, at $\delta > 0$, there will occur the excitation of acoustic vibrations which are caused by the phase transition and heat input to moist steam, with the increment

$$\lambda = \frac{2\pi\delta}{\sqrt{\omega_k^2 - \delta^2}}.$$

It further follows from formula (14) that, at $\delta > 0$ and $t \to \infty$, the amplitude of pressure oscillation infinitely increases. In reality, this does not happen because of nonlinear effects [6] and, as a result, low but finite amplitudes of oscillation obtain (a mode similar to vibration combustion). For determining these amplitudes, one must solve nonlinear equations which describe the self-oscillating process in view of the elastic properties of the channel and of the entire apparatus. However, this mode of operation is inadmissible for industrial facilities; therefore, a linearized system of equations is employed in the investigation of thermoacoustic instability, the solution of which enables one to determine the oscillation frequency and increment. Note that $p'/p \sim e^{\delta t}$ for other boundary conditions as well; therefore, it is of interest to investigate the conditions of excitation of acoustic vibrations as a function of the value of δ.. One can see that the expression for δ consists of two parts. The first part is defined by internal heat release due to phase transformations in moist steam, and the second part—by external heat input. In the absence of phase transitions $(dp/dy = 0; x^* = c_l T/L)$, the excitation of acoustic vibrations will be due to external heat input alone. We will consider qualitatively the conditions of excitation of acoustic vibrations without heat input. Under conditions of flow of moist steam in a channel $(dp/dy < 0)$, the excitation of acoustic vibrations due to phase transitions will occur at values of x in the range $1 > x > 0.7$. This is associated with the fact that, for moist steam, $x^* \approx 0.7$.. At $x < 0.7$, the excitation of acoustic vibrations will occur at $dp/dy > 0$, i.e., in the diffuser. The experimental verification of the results may consist in the following. Let moist condensing steam with $1 > x > 0.7$ and with $x < 0.7$ flow in the channel. In so doing, the condition $ka_S/2u \gg 1$, is valid, i.e., the time of isentropic process of moist steam is much longer than the period of acoustic vibrations. Then, according to my conclusions, acoustic vibrations must arise in the former case, and no acoustic vibrations – in the latter case. Experimental data on the excitation of acoustic vibrations under conditions of condensation of steam in a supersonic nozzle are given in [1]. Deich and Fillipov [1] explain the excitation of acoustic vibrations by self-vibratory motion in the nozzle of a pressure shock associated with the process of condensation. However, in the region of small variations of cross section at $1 > x > 0.7$, the expression for δ makes it possible to qualitatively explain the excitation of acoustic vibrations without considering pressure shocks. We will further consider the conditions in which $\delta = 0$.. In this case, the following expression may be derived:

$$q_l = -\frac{uR_0}{2x}\left(\frac{c_l T}{L} - x\right)\frac{dp}{dy}.$$

One can see that, with $x < 0.7$ and flow in the diffuser, $q_l < 0$. Therefore, heat removal is required for terminating the acoustic vibrations under these conditions. In the case of flow in the channel with $x < 0.7$, heat input is required. With $x > 0.7$ and flow in the diffuser, heat input is required; in the case of flow in the channel, heat removal is required.

In conclusion, we will give an example of calculation of q_l. Consider the flow in a channel. Let $dp/dy = -10^{-2}$ atm/m, $R_0 = 10^{-2}$ m, $x = 0.9$, and $u = 50$ m/s. Then, $q_l = -100$ W/m^2.

CONCLUSIONS

Expressions were derived for the frequency and increment of excitation of acoustic vibrations; in these expressions, the parameters of moist steam appear explicitly. The conditions of excitation of acoustic vibrations are given in this paper without invoking data on the size of droplets and on the processes of their interaction with environment. A formula was obtained for the density of external heat flux at which no acoustic vibrations arise.

REFERENCES

1. Deich, M.E. and Fillipov, G.A., *Gazodinamika dvukhfaznykh sred* (Gas Dynamics of Two-Phase Media), Moscow: Energoatomizdat, 1981.
2. Arutyunyan, G.M., *Termogidrodinamicheskaya teoriya geterogennykh sistem* (Thermohydrodynamic Theory of Heterogeneous Systems), Moscow: Nauka, 1994.
3. Pesochin, V.R., *Teplofiz. Vys. Temp.*, 2001, vol. 39, no. 3, p. 503 (*High Temp.* (Engl. transl.), vol. 39, no. 3, p. 470).
4. Pesochin, V.R., *Izv. Ross. Akad. Nauk Energ.*, 2008, no. 1, p. 145.
5. Linan, A. and Williams, F.A., *Fundamental Aspects of Combustion*, Oxford: University Press, 1993.
6. Artamonov, K.I., *Termogidroakusticheskaya ustoichivost'* (Thermohydroacoustic Stability), Moscow: Mashinostroenie, 1982.
7. Il'chenko, M.A., Kryutchenko, V.V., Mnatsakanyan, Yu.S., et al., *Ustoichivost' rabochego protsessa v dvigatelyakh letatel'nykh apparatov* (The Stability of the Working Process in the Engines of Flying Vehicles), Moscow: Mashinostroenie, 1995.
8. Kutateladze, S.S., *Osnovy teorii teploobmena* (Fundamentals of the Theory of Heat Transfer), Novosibirsk: Nauka, 1970.
9. Pesochin, V., R. *Teplofiz. Vys. Temp.*, 2008, vol. 46, no. 3, p. 471 (*High Temp.* (Engl. transl.), vol. 46, no. 3, p. 428).
10. Soo, S.L., *Fluid Dynamics of Multiphase Systems*, Waltham: Blaisdell, 1967. Translated under the title *Gidrodinamika mnogofaznykh sistem*, Moscow: Mir, 1971.